Sound

Mechanical pressure waves in air
(demo bell jar in vacuum)

Hearing: biological detection of sound waves

frequency: units $\frac{1}{\text{time}}$ $\Rightarrow$ $\frac{1}{s}$ = Hertz = Hz

freq range for hearing (approximate)

- low frequency: ~ 20Hz all ages
- highest " : depends on age

0-25 years old : up to ~ 20kHz

25-40 : up to ~ 15kHz

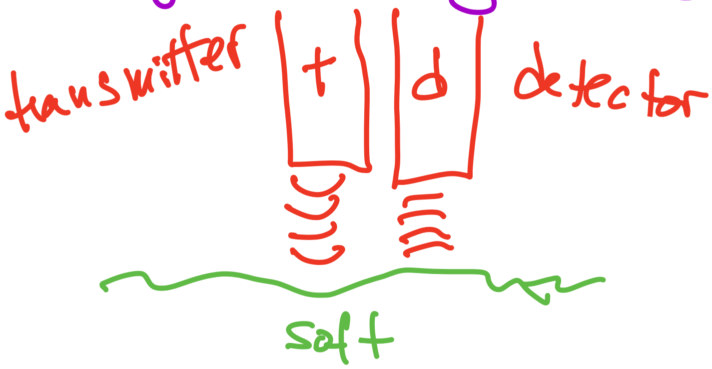
50+ : up to ~ 12kHz
(typically less for older people)

Ultrasound (US)
used for non-invasive measurements
(mostly in medicine)

$f_{US} \sim 2\text{-}15\ \text{MHz}$

can probe various densities

$\Rightarrow$ higher density $\rightarrow$ higher v_{sound} and longer echo



hard

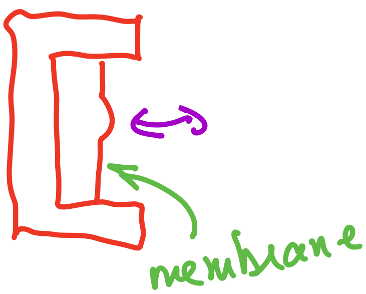
strength of echo depends on materials (soft, hard, etc)

$\Rightarrow$ measure: arrival time in detector

if you know velocities, you know density for different depths

Sound: compression & rarefaction of air

density ↗ density ↘



- membrane vibrates in/out
- pushes on air → creates longitudinal pressure

wave propagation out

- drive speaker with sine wave
- speaker moves in/out sinusoidally displacing air molecules:

$$s(x,t) = \underbrace{S_{max}}_{\text{amplitude = max displacement}} \cos(kx \pm \omega t)$$

note: energy from speaker $\Rightarrow$ air

- changing air displacement & pressure
- some heating of air

<u>velocity of sound</u> (m/s)

	air	water	glass	steel
v_s	~340	~1500	~5500	~6000

for air, $v = \sqrt{\dfrac{B}{\rho}}$

B = bulk modulus: measures resistance to compression under pressure so $B = \Delta P / \left(\frac{\Delta V}{V}\right)$ ΔP change in pressure

$\frac{\Delta V}{V}$ fractional change in volume

ρ = air density (kg/m^3) ~ 1.225 kg/m^3 at sea level, 68°F = 20°C

B and ρ change w/temp

so use v_0 = vel at $T = 20°C$

$$v = v_0 \sqrt{\frac{T}{273}} \qquad v_0 \sim 330 \text{ m/s}$$

T & 273 in K Kelvin

ex: v (T = freezing = 0°C = 273K = 330 m/s

20°C = 68F = 293 = 342 m/s

40°C = 104F = 313K = 353 m/s

$\sim 7\%$ higher

Derivation: see text

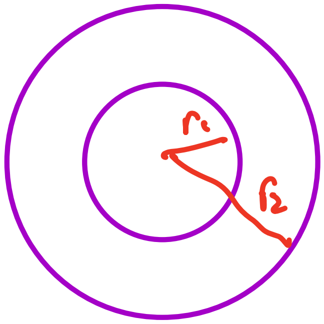
400 2/5

Intensity

same as Ch 16: $I = \frac{P_{av}}{\text{area}}$

sound travels in 3D, waves spread out over increasingly large area in 3D, area $= 4\pi r^2$ r is dist from source

units: $[I] = \frac{W}{m^2}$



Power is:

$P = IA$

integrate intensity over area

assume I is constant over area

then $P = I \cdot A = I \cdot 4\pi r^2$

P is what source puts out so is constant over any area

I goes down as $\frac{1}{r^2}$ as area increases by r^2

so at any r: $I_1 A_1 = I_2 A_2$

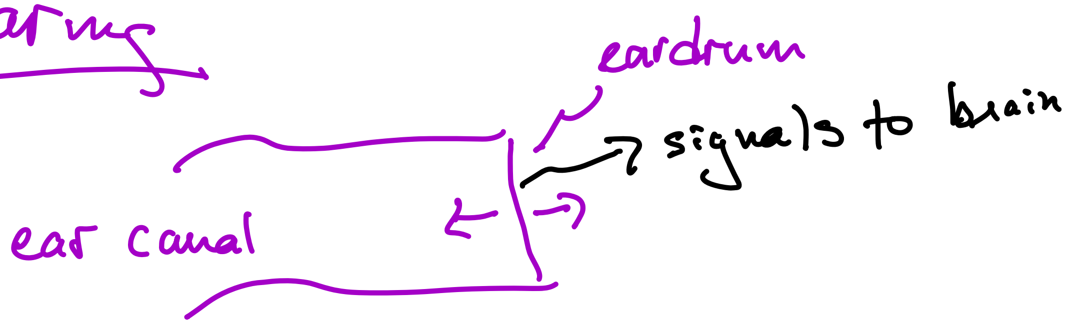
$$I_1 \cdot 4\pi r_1^2 = I_2 \cdot 4\pi r_2^2$$

or $I_2 = I_1 \cdot \frac{r_1^2}{r_2^2}$

so double r_1 intensity decreases by x4

200 2/7

Hearing



see any youtube video on how the ear works

Sound loudness

human ear handles huge range of intensities

threshold: $I_0 = 10^{-12}\ W/m^2$

pain level: $I_{max} = 1\ W/m^2$
$= 10^{12} I_0$

$\Rightarrow$ dynamic range of 10^{12}

decibel: $\beta(I) = 10 \log_{10}(I/I_0)$ (dB)

is another way of characterizing "loudness"

review logarithms

log: say $y = 10^x$
then $x = \log_{10} y$

1. $\log(x^a) = a \log x$
2. $\log 10 = 1$, $\log 1 = 0$
3. $\log(a \cdot b) = \log(a) + \log(b)$
4. $\log(a/b) = \log(a) - \log(b)$
5. $10^{\log_{10} x} = x$

dB for I_0:

$$\beta(I_0) = 10 \log(I_0/I_0) = 10\log(1) = 0 \text{ dB}$$

dB for I_{max}:

$$\beta(I_{max}) = 10 \log(10^{12} I_0/I_0) = 10\log(10^{12})$$
$$= 10 \cdot 12 = 120 \text{ dB}$$

dB	0	40	80	95	100	120
	threshold	sound level at home	loud radio	Temps b-ball	siren @ 30 m	loud rock concert

160 ⇒ eardrums burst

ex: Intensity I_1 is doubled to I_2
⇒ what is increase in dB?

$$\Delta\beta = 10\log(I_2/I_0) - 10\log(I_1/I_0)$$

$= 10 \left[\log(I_2/I_0) - \log I_1/I_0 \right] = 10 \log(I_2/I_1)$

$= 10 \log 2$ and $\log 2 = 0.30$

so $\Delta\beta = 3dB$ for $\times 2$ change in intensity

<u>What does intensity depend on?</u>

⇒ sound is a longitudinal wave

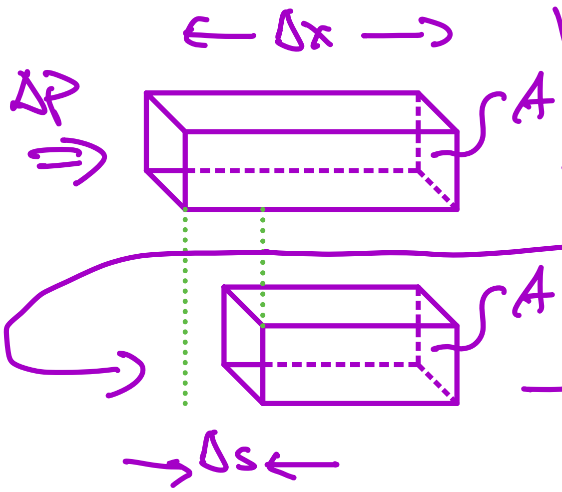
⇒ the <u>change</u> in pressure causes displacement

$s(x,t)$ is the displacement of molecules along x-direction

⇒ when pressure increases, molecules are squeezed together, no displacement yet due to inertia

⇒ molecules then move but stop when pressure gets small, turn around, etc.

⇒ this is the model of a longitudinal wave



$V = A\,\Delta x$

push on it with pressure ΔP

volume decrease

displacement is Δs

$\Delta V = A\,\Delta s$

note: Δp is increase of pressure above atmospheric

Bulk modulus: $\dfrac{\text{amount of pressure increase}}{\text{fractional change in volume}}$

$$B = -\frac{\Delta p}{\Delta V / V}$$

minus sign: as $\Delta p \nearrow$, $\Delta V \searrow$

$$\frac{\Delta V}{V} = \frac{A\,\Delta s}{A\,\Delta x} = \frac{\Delta s}{\Delta x} \to \frac{\partial s}{\partial x} \quad \text{as } \Delta x \to 0$$

200 2/10

from $B = -\dfrac{\Delta p}{\Delta V/V}$ write

$$\Delta p = -B\frac{\Delta V}{V} \xrightarrow[x\to 0]{} -B\frac{\partial s}{\partial x}$$

this is change in pressure needed to move the column (pressure increases by Δp) by amount Δs

use $s = s_{max}\cos(kx-\omega t)$ $(\phi = 0)$

then $\frac{\partial s}{\partial x} = -k s_{max}\sin(kx-\omega t)$

so $\Delta p = -B\frac{\partial s}{\partial x} = Bks_{max}\cos(kx-\omega t)$

or $\Delta p = \Delta P_{max}\cos(kx-\omega t)$

where $\Delta P_{max} = Bks_{max}$

$$\boxed{\begin{aligned} s &= s_{max}\cos(kx-\omega t) \\ \Delta p &= \Delta p_{max}\sin(kx-\omega t) \end{aligned}}$$

<u>out of phase</u>

and $\Delta p_{max} = Bks_{max}$

now use $v = \sqrt{B/\rho}$ vel sound so $B = v^2\rho$

substitute to get $\Delta p_{max} = v^2\rho k s_{max}$

$= v\cdot\frac{\omega}{k}\rho k s_{max}$

or

$$\boxed{\Delta P_{max} = v \omega \rho S_{max}}$$

Instantaneous intensity

Now calculate intensity $I(t) = \frac{\text{Power}(t)}{\text{area}}$

$P = \text{Power} = \frac{\Delta E}{\Delta t}$ and ΔE = work done

= Force * displacement

$= F \cdot \Delta S$

so $P = \frac{\Delta E}{\Delta t} = \frac{F \cdot \Delta S}{\Delta t} = F \cdot \frac{\Delta S}{\Delta t}$

and $\frac{F}{A} = \Delta p$ pressure

so $I = \frac{P}{A} = \frac{F \cdot \frac{\Delta S}{\Delta t}}{A} = \Delta p \cdot \frac{\Delta S}{\Delta t} \rightarrow \Delta p \cdot v_s$

and $v_s = \frac{\partial s}{\partial t} = \frac{\partial}{\partial t} S_{max} \cos(kx - \omega t)$

$= S_{max}\, \omega \sin(kx - \omega t)$

$I(t) = \Delta p(t) \cdot v_s(t)$ both depend on time

$= \Delta p_{max} \sin(kx - \omega t) \cdot \frac{\partial s}{\partial t}$

$= \Delta p_{max} \sin(kx - \omega t) \cdot S_{max}\, \omega \sin(kx - \omega t)$

$= \Delta P_{max} S_{max} \omega \sin^2(kx-\omega t)$

$= I_{max} \sin^2(kx-\omega t)$

where $I_{max} = \Delta P_{max} S_{max} \omega$

<u>Average intensity</u> $\bar{I}$

$\bar{I} = I(t)$ averaged over 1 cycle

so if $I(t) = I_{max} \sin^2(kx-\omega t)$ then

$\bar{I} = I_{max} \overline{\sin^2(kx-\omega t)} = \frac{1}{2} I_{max}$

because $\overline{\sin^2\theta} = \frac{1}{2}$ for any θ

so $\bar{I} = \frac{1}{2} I_{max} = \frac{1}{2} \Delta P_{max} S_{max} \omega$

substitute for S_{max} using $\Delta P_{max} = v\omega \rho S_{max}$

$\bar{I} = \frac{1}{2} \Delta P_{max} \omega \cdot \frac{\Delta P_{max}}{v\omega\rho} = \frac{\Delta P_{max}^2}{2v\rho}$

$$\boxed{\bar{I} = \frac{\Delta P_{max}^2}{2v\rho}}$$ average intensity (over 1 cycle)

ΔP_{max} = pressure amplitude of sound wave

ρ = density air & v = vel sound 400 2/10
= 1.225 kg/m³ = 330 m/s @ 0°C

ex: average intensity at hearing threshold:
$I_0 \sim 10^{-12} \frac{W}{m^2}$

so $I_0 = 10^{-12}\, W/m^2 = \frac{\Delta P_{max}^2}{2\rho v}$ solve for ΔP_{max}

$\Delta P_{max} = \sqrt{2\rho v I} = \sqrt{2 \cdot 1.225 * 330 * 10^{-12}}$

$= 28 \times 10^{-6}$ Pa very small pressure

find $S_{max} = \frac{\Delta P_{max}}{v \omega \rho}$ use $f = 1$ kHz

$S_{max} = \frac{20 \times 10^{-6}}{330 \cdot 2\pi \cdot 1.225 \cdot 1000} = \sim 10$ pm !!

pain threshold for hearing $I_{max} = 1\, W/m^2$

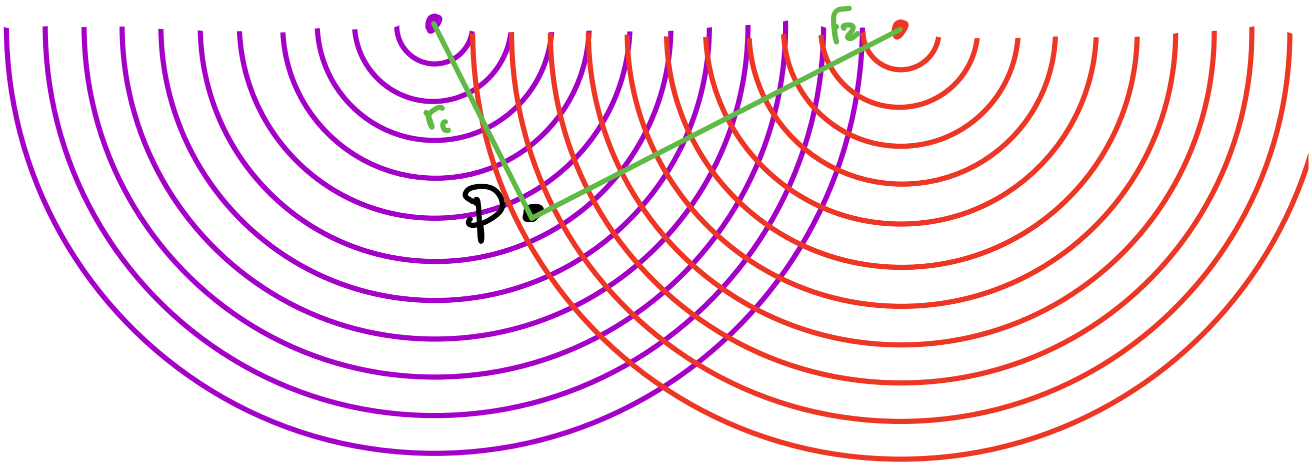
both ΔP_{max} and $S_{max} \sim \sqrt{I} \sim 10^6 \sqrt{I_0}$

so at pain theshold $\Delta P_{max} = 28$ Pa
$\Delta S_{max} = 10\,\mu m$

this shows how amazing hearing is!

200 2/14 Sound wave interference

- start w/ 2 sources of sound
- both have same amplitude & freq & velocity (so same λ)
- put sources in different locations



each wave has amplitude at point P given by

$$\left.\begin{aligned} y_1 &= A\sin(kr_1 - \omega t) \\ y_2 &= A\sin(kr_2 - \omega t) \end{aligned}\right\} \; r_1, r_2 \text{ is dist to P from each source}$$

add to get $y_{tot} = A\left[\sin(kr_1 - \omega t) + \sin(kr_2 - \omega t)\right]$

let $\Delta r \equiv r_2 - r_1$ difference

and $\bar{r} = \frac{1}{2}(r_1 + r_2)$ average solve for r_1, r_2: $\quad \Delta r = r_2 - r_1$

$$2\bar{r} = r_2 + r_1$$

so $2r_2 = \Delta r + 2\bar{r}$

$$r_2 = \bar{r} + \tfrac{1}{2}\Delta r$$

$$r_1 = \bar{r} - \tfrac{1}{2}\Delta r \quad \text{to get } r_2 - r_1 = \Delta r$$

so $\quad y_1 = A\sin(k\bar{r} - \tfrac{1}{2}k\Delta r - \omega t)$

$$y_2 = A\sin(k\bar{r} + \tfrac{1}{2}k\Delta r - \omega t)$$

$$\left.\begin{aligned} y_1 + y_2 &= A\sin(k\bar{r} - \omega t - \tfrac{1}{2}k\Delta r) \\ &+ A\sin(k\bar{r} - \omega t + \tfrac{1}{2}k\Delta r) \end{aligned}\right\} \text{let } k\bar{r} - \omega t \equiv \alpha$$

$$= A\sin(\alpha - \tfrac{1}{2}k\Delta r) + A\sin(\alpha + \tfrac{1}{2}k\Delta r)$$

$$= A(\sin\alpha\cos\tfrac{1}{2}k\Delta r - \cos\alpha\sin\tfrac{1}{2}k\Delta r)$$

$$+ A(\sin\alpha\cos\tfrac{1}{2}k\Delta r + \cos\alpha\sin\tfrac{1}{2}k\Delta r)$$

$$= 2A\cos\tfrac{1}{2}k\Delta r\sin\alpha$$

$$= 2A\cos(\tfrac{1}{2}k\Delta r)\sin(k\bar{r} - \omega t)$$

<u>Constructive interference condition</u>

if $\tfrac{1}{2}k\Delta r = 0, \pi, 2\pi, \ldots = n\pi$ then $\cos(\ ) = \pm 1$

so waves interfere constructively

so $\frac{1}{2} k \Delta r = n\pi$

$k \Delta r = n \cdot 2\pi$

$k = \frac{2\pi}{\lambda}$ so $\frac{2\pi}{\lambda} \Delta r = n \cdot 2\pi$ $n =$

$$\boxed{\Delta r = n\lambda, \; n = 0, 1, 2 \ldots \text{ constructive interference !}}$$

if path difference is $n\lambda \Rightarrow$ constructive interference

Destructive interference condition

if $\frac{1}{2} k \Delta r = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$ then $\cos(\;) = 0$ destructive

$\frac{1}{2} \frac{2\pi}{\lambda} \cdot \Delta r = \frac{1}{2} n\pi$, $n = 1, 3, 5$ odd

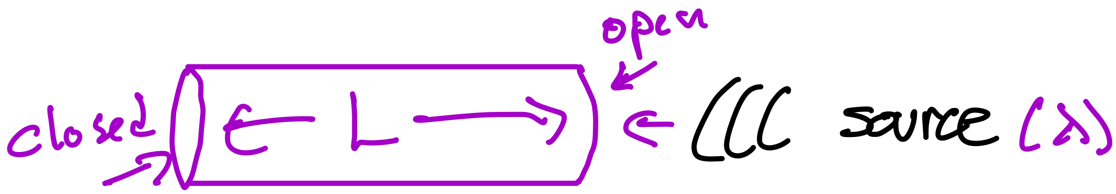
$\Delta r = \frac{1}{2} n\lambda$ $n = $ odd

or $\Delta r = \frac{1}{2}(2n+1)\lambda$, $n = 0, 1, 2 \ldots$

$$\boxed{\Delta r = (n + \tfrac{1}{2})\lambda \quad n = 0, 1, 2 \ldots \text{ destructive}}$$

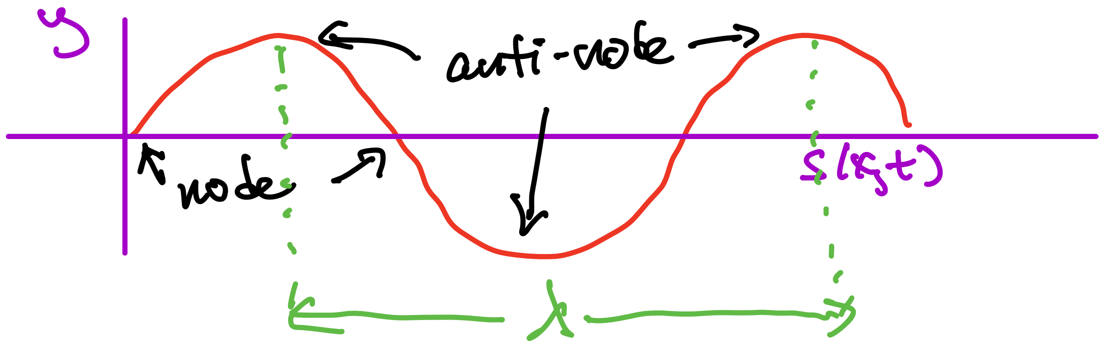
see https://www.physics.umd.edu/hep/drew/optics/antenna.html

Resonance in tube w/ one end closed



- molecules of air cannot move at closed end but can at open end
- so if displacement wave has
  node (no movement) at closed end
  anti-node (max movement) at open end

this wave will resonate in tube



so if source emits sound w/ wavelength:

$L = \frac{1}{4}\lambda$

then reflected wave & incoming wave will interfere constructively and cause resonance also $L = \frac{3}{4}\lambda$, and others adding $\frac{1}{2}\lambda$

resonance at $L = \frac{1}{4}\lambda, \frac{3}{4}\lambda, \frac{5}{4}\lambda, \frac{7}{4}\lambda, \ldots$

or $\lambda_n = 4L, \frac{4L}{3}, \frac{4L}{5} \ldots = \frac{4L}{n}$ $n = 1, 3, 5 \ldots$

only these will "fit" and resonate

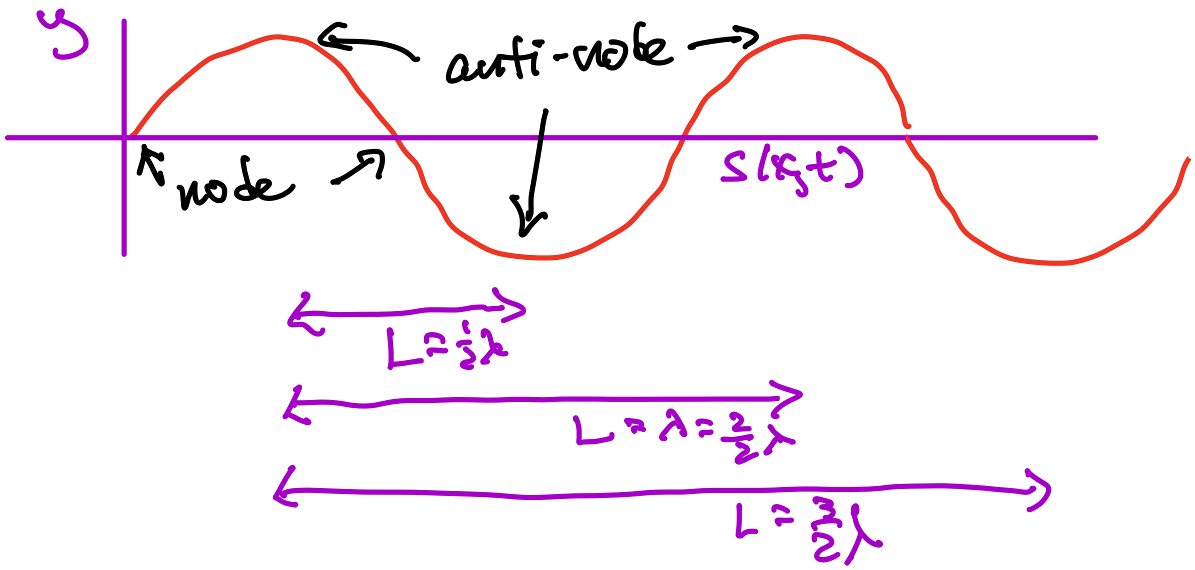
frequency: $v = \lambda f$ so $f = \frac{v}{\lambda}$

$f_n = \frac{v}{4L/n} = n\frac{v}{4L}$, $n = 1, 3, 5 \ldots$

$f_1 = \frac{v}{4L}$ so $f_n = n f_1$ f_1 fundamental frequency

Tube open at both ends

$\Rightarrow$ resonates at wavelengths that have anti-nodes at both ends



so $L = \frac{n\lambda}{2}$ $n = 1, 2, 3 \ldots$

or $\lambda_n = \frac{2L}{n}$ "

$f_n = \frac{v}{\lambda_n} = \frac{nv}{2L} = nf_1$, $n = 1, 2, 3 \ldots$

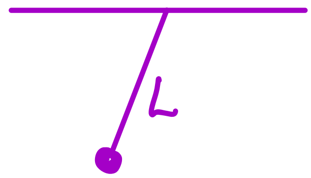
$f_1 = \frac{v}{2L}$

Resonance

1. any mechanical or electrical system will have "natural oscillation"

ex: • child on swing

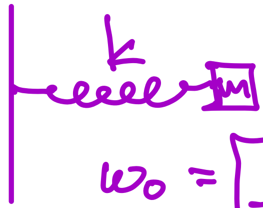
- pendulum



$\omega_0 = \sqrt{g/L}$ $\qquad \omega_0 = 2\pi f_0$

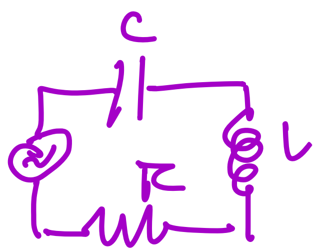
f_0 is "natural" freq

- spring

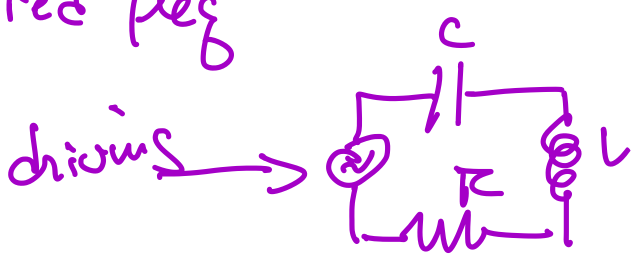


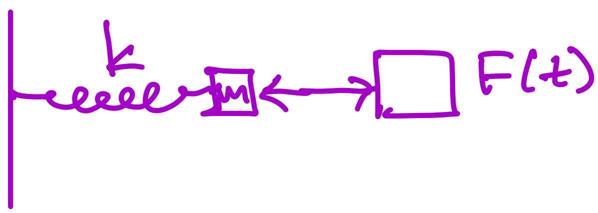
$\omega_0 = \sqrt{\frac{k}{m}}$

- LCR circuit



2. you can "drive" any system at any desired freq





if driving frequency matches natural frequency $\Rightarrow$ resonance

Beats

2 tuning forks w/ different $\omega = 2\pi f$

$y_1 = A \sin(\omega_1 t)$

$y_2 = A \sin(\omega_2 t)$

add: $y_{tot} = A \sin(\omega_1 t) + A \sin(\omega_2 t)$

$= A [\sin(\omega_1 t) + \sin(\omega_2 t)]$

form $\bar{\omega} \equiv \frac{\omega_1 + \omega_2}{2}$

$\Delta\omega \equiv \omega_1 - \omega_2$

solve for ω_1, ω_2

$2\bar{\omega} = \omega_1 + \omega_2$

$\Delta\omega = \omega_1 - \omega_2$

add: $2\bar{\omega} + \Delta\omega = 2\omega_1$

$\omega_1 = \bar{\omega} + \frac{1}{2}\Delta\omega$

$\omega_2 = \omega_1 - \Delta\omega = \bar{\omega} - \frac{1}{2}\Delta\omega$

rewrite: $y_{tot} = A [\sin(\bar{\omega} t + \frac{1}{2}\Delta\omega t) +$

$\sin(\bar{\omega} t - \frac{1}{2}\Delta\omega t)]$

expand $\sin(\bar{\omega} t \pm \frac{1}{2}\Delta\omega t)$

$$= \sin\bar{\omega}t \cos\tfrac{1}{2}\Delta\omega t \pm \cos\bar{\omega}t \sin\tfrac{1}{2}\Delta\omega t$$

$$y_{tot} = A\left[\sin\bar{\omega}t \cos\tfrac{1}{2}\Delta\omega t + \cos\bar{\omega}t \sin\tfrac{1}{2}\Delta\omega t + \sin\bar{\omega}t \cos\tfrac{1}{2}\Delta\omega t - \cos\bar{\omega}t \sin\tfrac{1}{2}\Delta\omega t\right]$$

$$= 2A\cos(\tfrac{1}{2}\Delta\omega t)\sin(\bar{\omega}t)$$

if f_1 & f_2 are "close", then $\Delta\omega \ll \bar{\omega}$

ex: $f_1 = 440\ Hz$

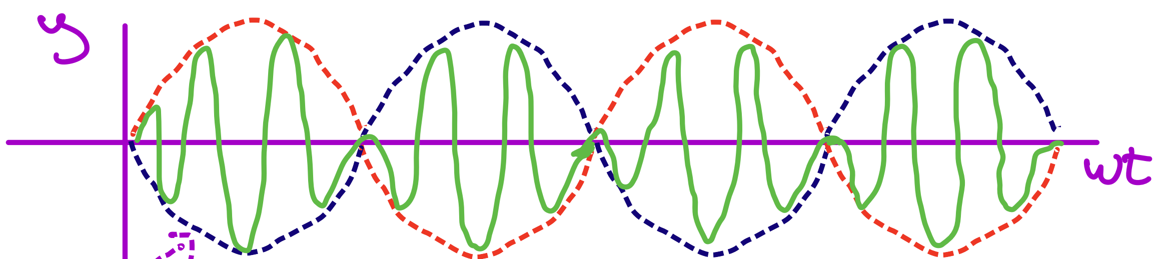
$f_2 = 442\ Hz$

$\Delta f = 2 Hz$

$\bar{f} = 441\ Hz$

$2A\cos(\tfrac{1}{2}\Delta\omega t)$ will be like a very slowly varying amplitude of a sound at frequency $\bar{f}$

see https://www.physics.umd.edu/hep/drew/superposition.html



$A\cos(\tfrac{1}{2}\Delta\omega \cdot t)$

The signal $\sin(\bar{\omega} t)$ is "modulated"
by $\frac{1}{2}\Delta\omega t$ → "Beat freq" is $|\Delta f|$

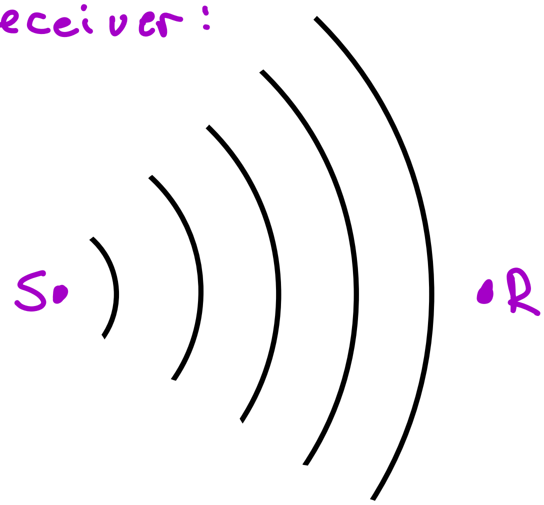
Doppler Effect

see "non Relativistic Doppler Shift" simulation

"normal" - stationary source and receiver

→ what happens when source moves?
e.g. police car w/siren driving by

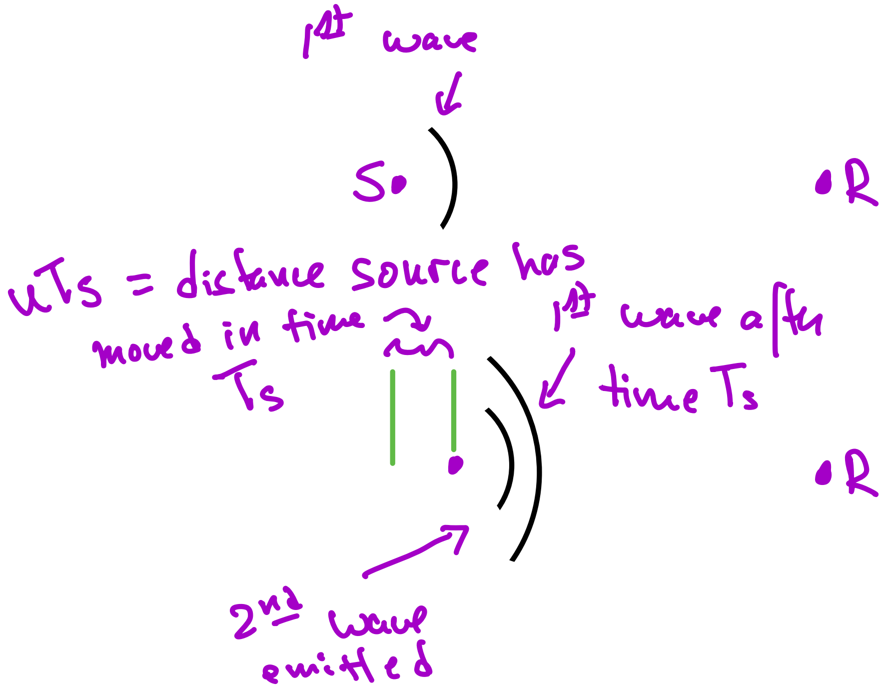
stationary source & receiver:



Source moves w/velocity u
Wave moves w/velocity $v = \lambda_s f_s$ and $f_s = 1/T_s$

- source puts out wave, and after time T:
  - wave is distance λ away
- during that time source has moved closer to receiver

- next wave will be emitted closer to receiver than 1st wave



- receiver R sees smaller dist d between waves

$d = \lambda_s - u\Delta t$ $\quad$ Δt is time between wave emission: $\Delta t = T_s$

so $d = \lambda_s - uT_s$ $\quad$ use $v = \lambda f_s$, $f_s = 1/T_s$

$$= \frac{v}{f_s} - \frac{u}{f_s} = \frac{v-u}{f_s}$$

R sees waves arriving that have wavelength $\lambda_R = d$ and waves are still moving w/velocity v and $v = \lambda f$

so $v = \lambda_R f_R = d f_R$

$d = \frac{v}{f_R}$

so $d = \lambda_S - u T_S$

or $\lambda_R = \lambda_S - \frac{u}{f_S}$ us $\lambda_R = \frac{v}{f_R}$

$\frac{v}{f_R} = \frac{v}{f_S} - \frac{u}{f_S}$

$\frac{f_R}{v} = \frac{f_S}{v-u}$ source moving towards R

$\boxed{f_R = f_S \frac{v}{v-u}}$ S moving towards R

if S moves away from R, $u \to -u$

$\boxed{f_R = f_S \frac{v}{v+u}}$ S moving away from R

What if receiver is moving?

moving receiver

- dist between waves also decreases

$d = \lambda_S - u\Delta t$ but now Δt is period of wave that R measures

$\Delta t = T_R$

so $d = \lambda_S - uT_R$

and $d = \lambda_R$ (just like before) dist between waves that R measures

so $\lambda_R = \lambda_S - uT_R$

$$= \lambda_S - \frac{u}{f_R}$$

use $v = \lambda_R f_R = \lambda_S f_S$

so $\frac{v}{f_R} = \frac{v}{f_S} - \frac{u}{f_R}$

$\frac{v+u}{f_R} = \frac{v}{f_S} \Rightarrow \boxed{f_R = f_S \frac{v+u}{v}}$ R moving towards S

for R moving away from S, $u \to -u$

$$\boxed{f_R = f_S \frac{v-u}{v}}$$ R moving away from S

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